Creep of superconducting vortices in the limit of vanishing temperature: A fingerprint of off-equilibrium dynamics.

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We theoretically study the creep of vortex matter in superconductors. The low temperatures experimental phenomenology, previously interpreted in terms of "quantum tunnelling" of vortices, is reproduced by Monte Carlo simulations of a purely "classical" vortex model. We demonstrate that a non-zero creep rate in the limit of vanishing temperature is to be expected in systems with slow relaxations as a consequence of their off-equilibrium evolution in a complex free energy landscape.

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There exist an abundance of experimental evidence that the relaxation rate of the magnetisation in type II superconductors (ranging from conventional, to organic, heavy fermions and a variety of $H-T_c$ superconductors [1–3]) does not vanish as the temperature T is lowered towards zero (see eg. [1–3]). This may seem surprising if one assumes that the mechanism allowing the magnetisation to relax is thermal activation over a characteristic energy barrier ΔE. Namely, when $T \ll \Delta E$ the relaxation rate should vanish since the Arrhenius factor for thermal activation, $\exp(-\Delta E/T)$, goes to zero. The question then arises how relaxation can take place at a finite rate while the thermal activation factor exponentially approaches zero.

A number of researchers have suggested that such a phenomenon is caused by quantum tunnelling of vortices through the barriers in the random pinning potential (for a theoretical review see [4]). The above "quantum" explanation is very intriguing and in good agreement with some experimental results in compounds such as YBCO [1,2] or BSCCO [5]. Also other even more exotic materials, such as organic superconductors [6], give good correspondences. However, in other non conventional systems such as heavy fermion superconductors, the theory of quantum creep is totally unable to describe the observed low T relaxation [7]. In fact, strong discrepancies are found in many other systems ranging from PCeCO crystals to YBCO/PBCO multilayers or YBCO and BSCCO films and crystals [8]. One problem is [8] that the length of the tunnelling vortex segment $L_c(0)$ needed to fit the creep rate data, S, can be orders of magnitude larger that the one theoretically predicted by quantum creep theory [4]. Also the experimental temperature dependence of the creep rate, S, is often very different from the one predicted by quantum theory (see [5]).

The above contradictory results suggests to look for additional descriptions of the anomalous low T magnetic relaxation. It is worth to stress that the observation of a non-vanishing constant creep rate in the limit $T \to 0$ is found under very general circumstances: it does not crucially depend on the thickness of the sample [8] (i.e. on its dimensionality), nor on whether the pinning is caused by columnar defects or random point pins [9]. Thus, the mechanism behind the low temperature creep seems to be of a fundamental and basic nature.

We demonstrate below that also in a "classical" system (i.e., not "quantum") logarithmically slow glassy dynamics can naturally persist even at vanishing temperatures and can lead to the experimentally observed phenomenology. This is possible because the low T off-equilibrium dynamics consists of searching, among a very large number, for a few "downhill" or "flat" directions in the free energy landscape. The number of these directions decreases as relaxation proceeds though there always remain some. They can be found only by collective cooperative rearrangements of the system, resulting in a slowing down of relaxation [10].

The model – We study a statistical mechanics model for vortex matter called a Restricted Occupancy Model (ROM) [11]. In the limit of zero temperature and infinite upper critical field, it reduces to a cellular automaton introduced in [12] to study vortex avalanches. We use Monte Carlo dynamics which enabled the ROM to depict a unified picture of creep and transport phenomena in vortex physics, ranging from magnetisation loops with "anomalous" second peak, logarithmic relaxation, Bean profiles, to history dependent behaviours in vortex flow and I-V characteristics, to the reentrant nature of the equilibrium phase diagram [11]. The model also predicts the existence of a "glassy region" at low temperature with strong "aging" effects [11].

Here we use the ROM to study the magnetic relaxation rate, S, in the very low temperature limit. Interestingly, the model reproduces the experimental "anomalous" relaxation and the observed behaviour of S [1–3,8].

The ROM model is described in full details in Ref. [11]. A system of straight parallel vortex lines is coarse grained in the xy-plane by introducing a square grid of lattice spacing l_0 of the order of the London penetration length, λ [11]. The number of vortices on the i-th coarse grained plaquette is denoted by n_i. The occupancy of each plaquette is a number larger than zero and, importantly, smaller than $N_{c2} = B_{c2} l_0^2/\phi_0$, where B_{c2} is the upper critical magnetic field and $\phi_0 = hc/2e$ is the magnetic flux quantum. The ROM model is thus defined by the following coarse grained vortex interaction Hamiltonian [11]: $\mathcal{H} = \frac{1}{2}\sum_{ij} n_i A_{ij} n_j - \frac{1}{2}\sum_i A_{ii} n_i - \sum_i A_i^p n_i$. The first two terms describe the repulsion between the

vortices and their self energy. On-site and nearest neighbour interactions are included, i.e., the interaction between vortex lines with a separation greater than the London screening length (which by definition is close to l_0) is ignored. We choose $A_{ii} = A_0 = 1$; $A_{ij} = A_1$ if i and j are nearest neighbours and $A_{ij} = 0$ otherwise. The last term in $\mathcal{H}$ describes a delta-distributed random pinning $P(A^p) = (1-p)\delta(A^p) + p\delta(A^p - A_0^p)$. Interestingly, the general scenario of creep phenomena we describe below does not depend on the details of pinning in the system (a fact in correspondence with experimental results [8]). In our model A_0 sets the energy scale. Below we choose $A_0 = 1.0$; $A_0^p = 0.3$; $N_{c2} = 27$; $p = 1/2$; $\kappa^* \equiv A_1/A_0 \in [0.26, 0.3]$.

The relaxation of the model is simulated by use of Monte Carlo dynamics on a square lattice in presence of a thermal bath of temperature T. The system is periodic in the y-direction. The two edges parallel to the y-direction are in contact with a vortex reservoir. Particles can enter and exit the system only through the reservoir, which plays the role of the external magnetic field. Hence the reservoir density, N_{ext}, is used as the external control parameter. We perform the following zero field cooled experiment: at a low temperature T we increase at constant rate $\gamma = \Delta N_0/\tau$ the reservoir density from zero to a working value N_{ext}. We keep N_{ext} fixed while we monitor the time dependence of the magnetisation $M = N_{in} - N_{ext}$. Here $N_{in} = \sum_i n_i/L^2$ is the vortex density inside the system (of size L^2 [13]). Time is measured in units of single attempted updates per degrees of freedom of the lattice (see Ref. [14]). The data presented below are averaged over 128 realizations of the pinning background.

In particular, we investigate the creep rate

$$S = \left| \frac{\partial \ln(M)}{\partial \ln(t)} \right| \tag{1}$$

as function of T, N_{ext} and γ. In typical experiments the nature of the t dependence of M is such that S decreases in time. So usually, one deals with an average creep rate, S_a, in some given temporal window [1–7]. As shown in the upper inset of Fig.2, in our model dynamics $M(t)$ at low temperatures behaves according to the known logarithmic interpolation formula (see Ref. [11]) found in experiments [4], namely: $M(t) - M(0) \simeq \Delta M_\infty\{1 - [1 + \frac{\mu T}{U_c}\ln(\frac{t+t_0}{t_0})]^{-1/\mu}\}$. Here, $M(0)$ is the magnetisation at the time of preparation of the sample ($t = 0$), ΔM_∞ its overall variation, the exponent μ is consistent with 1, $\frac{\mu T}{U_c}$ and t_0 are fit parameters [4].

Consistently, we define S_a as the average value of S in the last time decade of our measures (i.e., for $t \in [10^5, 10^6]$). The present choice, analogous to those made for experimental data, is the most natural one and the general results presented below do not depend on it.

For the reasons explained in the introduction, a very important physical quantity is the distribution, $P(\Delta E)$, of the energy barriers, ΔE, that vortices segments meet during their motion. Since at low T the system is typically off-equilibrium, $P(\Delta E)$ is itself a (logarithmically slow) function of t and we consider its average over the last time decade of our measurements.

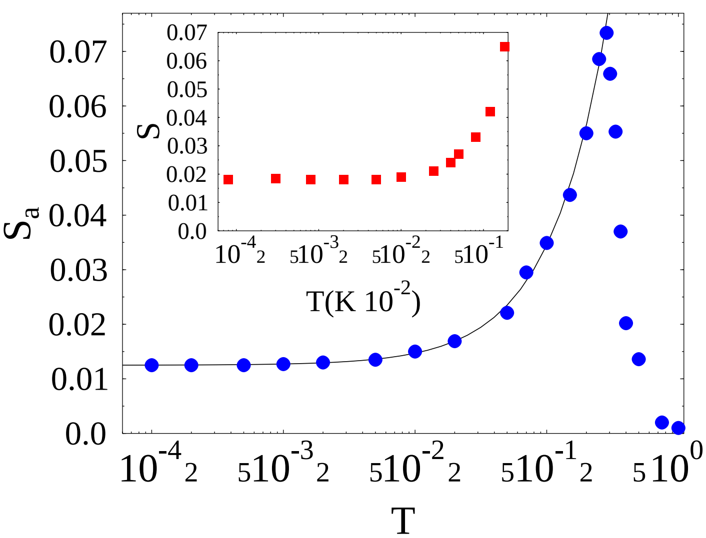

FIG. 1. **Main frame** The creep rate, S_a, in the ROM model ($\kappa^* = 0.28$, $\gamma = 10^{-3}$) for $N_{ext} = 10$ as a function of the temperature, T, in units of A_0. The error bars are of the size of symbols. The superimposed line is a linear fit. **Inset** The creep rate measured in BSCCO single crystal from Aupke et al. [5] for an external field of 880 Oe.

Results – When the temperature is very low, the model exhibits the same kind of "anomalous" creep found in the experiments on superconductors. In Fig.1, we plot the creep rate, S_a, as a function of T in a broad temperature range. For comparison we present equivalent experimental measurements in a BSCCO single crystal (from Ref. [5]) as inset. The numerical values found for S_a at low T in our model and in real samples are interestingly very similar. The temperature scales of the simulations and of real experiments can be compared by considering that the ratio T/A_0 in our model is of the same order of magnitude as T/T_c in a real superconductor. This is seen from a comparison of the (T, N_{ext}) equilibrium phase diagram of our model with the equilibrium temperature-magnetic field, (T, H), phase diagram of, say, a BSCCO superconductor (see Ref. [11]).

In both experiments and simulations, S_a approaches a finite value, S_a^0, when $T \to 0$. In particular, we find that a linear fit of $S_a(T)$ in the low T regime is very satisfactory (see Fig.1):

$$S_a(T) = S_a^0 + \sigma T \tag{2}$$

where both S_a^0 and σ are a function of the applied field N_{ext}. We also note that in the present model $S_a(T)$ is non monotonous in T: in the higher T region it starts decreasing. This is also a known experimental fact [6,15], we discuss it later on. The maximum in $S_a(T)$ is just above

a characteristic crossover "glassy" temperature, T_g, defined in [11].

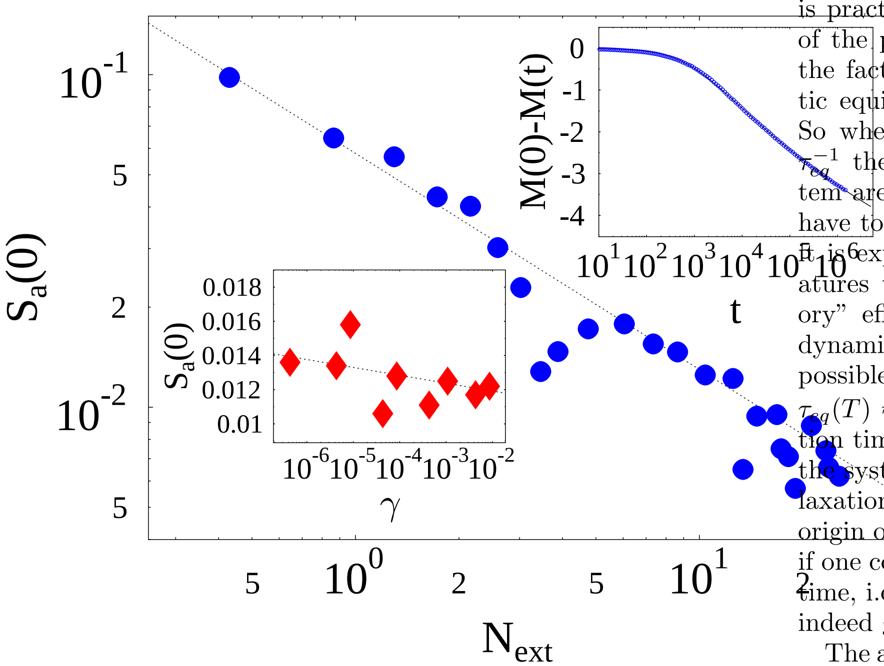

FIG. 2. **Main frame** The zero temperature limit of the creep rate, S_a, in the ROM model as a function of the applied field N_{ext} (for $T = 10^{-4}$ and $\gamma = 10^{-3}$). The superimposed curve is a power law to guide the eye. **Inset top:** The relaxation of the magnetisation, $M(t)$, in the model ($\kappa^* = 0.28$, $\gamma = 10^{-3}$) for $N_{ext} = 10$ and $T = 0.25$ as a function of time. The continuous line is the logarithmic fit of the text. **Inset bottom:** S_a is here plotted as a function of the sweep rate, γ, of the external field for $N_{ext} = 10$ and $T = 10^{-4}$. A very weak dependence is found.

In our model by varying the applied field we find a range of values for S_a^0 very similar to experimental ones [1–7] (see Fig.2). In particular, S_a^0 seems to decrease on average by increasing the field N_{ext}. The overall behaviour can be approximately interpolated with a power law: $S_a^0(N_{ext}) \simeq (N_{ext}/N_0)^{-x}$, where, for $\kappa^* = 0.28$, $N_0 \simeq 0.01$ and $x \simeq 0.6$. As shown in Fig.2, the presence of a small exponent x implies that sensible variations in S_0^a can be seen only by changing N_{ext} of orders of magnitude. Note that the dips in the $S_a(0)$ versus N_{ext} data in Fig. 2 at certain values of N_{ext} (namely around 3, 13, and 18) are statistically significant. They are related to the low field order-disorder transition, the 2nd peak transition and the reentrant high field order-disorder transition respectively [11].

In the lower inset of Fig.2, we show that S_a is essentially independent of the ramping rate, γ (the values shown are for $N_{ext} = 10$ and $T = 10^{-4}$). This is an other typical experimental observation [1]. However, a very small decrease of S_a with increasing γ cannot be excluded: we show a fit to the form $S_a^0 \simeq S_1 + s_2 \ln(\gamma)$, with $S_1 = 0.11$ and $s_2 = -2 \cdot 10^{-4}$. The fact that S_a^0 is practically independent on γ, far from being a proof of the presence of equilibrium in the system, is due to the fact that at very low temperatures the characteristic equilibration time, τ_{eq}, is enormous (see Ref. [11]). So whenever the driving rate, γ, is much larger than τ_{eq}^{-1} the off-equilibrium state and dynamics of the system are essentially independent of γ. Stronger γ effects have to be expected when γ gets closer to τ_{eq}^{-1}. In fact, it is experimentally well known that at higher temperatures the systems exhibit strong γ dependent "memory" effects [11,15,16], the signature of off-equilibrium dynamics. Actually, in the present model at low T, it is possible to show that $\tau_{eq}(T)$ diverges exponentially [11]: $\tau_{eq}(T) \sim \exp\left(\frac{E_0}{T}\right)$. In that region, the typical observation time windows, t_{obs}, are such that $t_{obs}/\tau_{eq} \ll 1$, and the system is in the early stage of its off-equilibrium relaxation from its initial state. This is schematically the origin of the flattening of S_a at very low T. Notice that, if one could observe the system for an exponentially long time, i.e., if $t_{obs}/\tau_{eq} \gg 1$, then the creep rate, S_a, would indeed go to zero.

The above scenario is clarified by the analysis of the energy barrier distribution function, $P(\Delta E)$, recorded during the system evolution at very low T. Such a quantity also clearly shows the simple mechanical origin of the anomalous creep found at very low temperature in the present model. The function $P(\Delta E)$ (where ΔE is in units of A_0), recorded at $T = 10^{-4}$, is plotted in Fig.3 for two values of the applied field, N_{ext}. We always find that $P(\Delta E)$ has support also on the negative axis. This is the mark of the off-equilibrium nature of the evolution on the observed time scales. The presence of a $P(\Delta E)$ which extends down to negative values also explains the presence of the recorded relaxation at low T: in the configuration space the system can still find directions where no positive barriers have to be crossed. The insert in Fig. 3 clarifies the mechanism behind the relaxation. Here, we plot the signal $A(t)$ defined, for each single Monte Carlo (MC) step t, in the following way: $A(t) = 0$ if the MC trial is rejected; $A(t) = 1$ if the trial is accepted and the energy reduced, i.e. $\Delta E \leq 0$; and finally $A(t) = 2$ when a trial is accepted with $\Delta E > 0$. We plot two sequences of trials. One for $0 \leq t \leq 500$ was measured at the early stage of the relaxation, the second sequence, placed at the interval $500 \leq t \leq 1000$, is measured at the late stage of the relaxation. Most trials are rejected ($A(t) = 0$) and only once in a while the system does manage to find a route pointing downhill in the energy landscape. $A(t) = 2$ never occurs. As time proceeds fewer and fewer "negative channels" are available to the relaxation and a decrease in the density of the spikes in $A(t)$ is observed.

As the temperature is increased thermal activation over positive energy barriers will become possible as the

Arrhenius factor $\exp(-\Delta E/T)$ assumes a non-vanishing value for an appreciable range of barrier values $\Delta E < T$. When this happens the relaxation will occur sufficiently fast to allow one, within the experimental time window, to closely approach the equilibrium configurations where the vortex density profile is more or less flat and relaxation ceases, hence S_a goes down, as seen in experiments [6,15] and in Fig. 1.

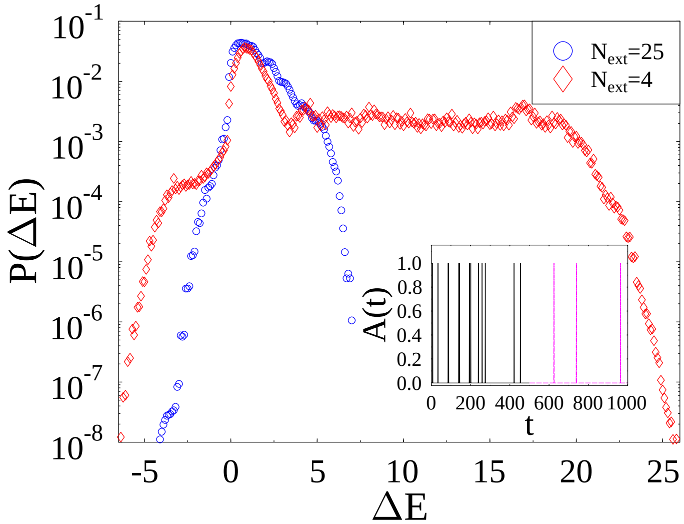

FIG. 3. The energy barrier distribution, $P(\Delta E)$, recorded at $T = 10^{-4}$ for $N_{ext} = 25, 4$ (resp. circles,diamonds). Notice that $P(\Delta E)$ extends down to negative values. **Inset** We plot the function $A(t)$ defined in the text to monitor the activity during relaxation. In the interval $[0, 500]$ we show $A(t)$ recorded at the beginning of the run and in $[500, 1000]$ $A(t)$ in the last steps of the same run.

Finally, we stress that slow off-equilibrium relaxations at very low temperatures are also observed in glass forming liquids [10]. In that cases too, no activation over barriers occurs and the system simply wanders in its very high dimensional phase space through the few channels where no energy increase is required.

Discussion - We have above demonstrated that the phenomenological behaviour of the creep rate at low temperatures can be understood in terms of the off-equilibrium nature of the inhomogeneous vortex density profile produced in magnetic creep experiments. In fact, cooperative mechanical rearrangements, possible even at very low T (where thermal activation over positive barriers can be negligible), dominate the phenomenon [10]. In this perspective, it is very important to stress that the system's equilibration time at very low temperature is much larger than any experimentally accessible time window [11]. Accordingly, we can say that experimental findings do not enforce the interpretation in terms of macroscopic quantum tunnelling of vortices.

Relaxation due to quantum tunnelling might be present along with the mechanical relaxation discussed above. It is then important to ask how compelling the quantum tunnelling interpretation is. The theory of quantum tunnelling assumes a London picture and treats the position of the vortex core as the variable that is able to tunnel. It is not entirely clear if this is the right level at which to introduce quantum fluctuations, but, more importantly, the quantum tunnelling description assume the existence of a characteristic energy barrier [4], not a time dependent distribution of barrier heights as typically found in many-body systems relaxing off-equilibrium. Finally, the quantum tunnelling description also tacitly assumes the existence of a static equilibrium state in which barriers are always positive. As we have clearly shown above, this is typically not the case and a dynamical approach is more appropriate.

The present scenario, where off-equilibrium phenomena dominate the anomalous low T creep, could be experimentally verified by the discovery of "aging" phenomena [11] like those recently observed in [17,18].

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